A Breif Description of the Pigeonhole Principle

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August 28th 2023

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1 Pigeonhole Principle

The Pigeonhole Principle is a mathematical principle used in Discrete math that states that if n items are put into m containers, when $n > m$, then at least one container must contain more than one item.

ex. If 3 pigeons (items) are put into 2 holes (containers) then one of the pigeons must go into the same hole as another pigeon.

Seems easy enough, however, why don't we start to apply some math to this concept. In Discrete Math we use things called Sets. Now a set is just a collection of elements, and elements belonging to the set are called members. The important thing to remember is that all the members of a set are distinct or unique. So each of the members are different, and in a set the order of the list of elements doesn't matter.

Now that we understand what sets are we can discuss what a Finite set is. Finite sets are sets that can in principle be listed as full. The thing about Finite sets is they hace cardinality, that is shown by using,$_{||}$.

For example lets say D represents the days of the week so the set D would equal Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday. To put this mathmatically.

$$D = Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday$$

So applying this to a Finite set we can say that:

$$|D| = 7$$

Or the Cardinality of D in this Sequence is 7.

Now that we understand sets a little more we can start talking about Functions. A Function from one set to another is a rule that will associates each member of the first set with exactly one member in the second set. To say that in some more math terms if f is a function from x to y and $x \in X$ then $f(x)$ is the member of Y that function associates with x. We call these functions mapping as we're mapping elements to other elements.

Lets walk through a problem that will tie all of this together:

$$f : X \to Y, |X| > |Y| \Rightarrow \exists x_1, x_2 \in X : x_1 \neq x_2 \wedge f(x_1) = f(x_2)$$

Now if we walk through this $f : X \to Y$ means that f is mappinf from the set X to the set Y. Next we have $|X| > |Y|$ This just tells us that the cardinality of the set X is greater than the cardinality of set Y. $\Rightarrow$ Just means implies. And $\exists$ simply means there exists so since the first part is true this **Implies There exists**. $x_1, x_2 \in X$: These just represent the members of X. $x_1 \neq x_2$ This represnts that these two values are not equal to each other. $\wedge$ is Just a fancy symbol for the word And. Finally, $f(x_1) = f(x_2)$ means that both of those numbers go through the same Y. There you go! you've successfully worked through your first problem in Discrete Math using the Pigeonhole Principle!

2 Extended Pigeonhole Principle

Now that you know the basics of the Pigeonhole theory lets explore some more interesting concepts. Lets start With Floors and ceilings. First lets talk about floors, If x is any real number, we write $\lfloor x \rfloor$ for the greatest integer less than or equal to x. So lets have an exampe of what that looks like.

$$Ex.\lfloor 17.8 \rfloor = 17$$

The value when the floor is applied will round down to the nearest integer no matter the case. Lets apply this to a fraction.

$$\lfloor \tfrac{1{,}000{,}017}{1{,}000{,}019} \rfloor \neq .9999 \text{ but instead} = 0 \text{ because the integer is rounded down.}$$

Now we take this principle and flip it to discuss ceilings, a ceiling is just the inverse of the floor so while the floor rounds a number down to the nearest integer, it rounds the number up to the nearest integer. For example:

$$\lceil 4.000001 \rceil = 5$$

Its essentially the same priciple as the floor, and now that you understand both of those, you have all the basic knowledge over the PigeonHole Principle.